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ANALYTICAL AND NUMERICAL INVESTIGATION OF ANT BEHAVIOR UNDER CROWDED CONDITIONS

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Swarm intelligence is widely recognized as a powerful paradigm of self-organized optimization, with numerous examples of successful applications in distributed artificial intelligence. However, the role of physical interactions in the organization of traffic flows in ants under crowded conditions has only been studied very recently. The related results suggest new ways of congestion control and simple algorithms for optimal resource usage based on local interactions and, therefore, decentralized control concepts. Here, we present a mathematical analysis of such a concept for an experiment with two alternative ways with limited capacities between a food source and the nest of an ant colony. Moreover, we carry out microscopic computer simulations for generalized setups, in which ants have more alternatives or the alternative ways are of different lengths. In this way and by variation of interaction parameters, we can get a better idea, how powerful congestion control based on local repulsive interactions may be. Finally, we will discuss potential applications of this design principle to routing in traffic or data networks and machine usage in supply systems.

Keywords: Ants; traffic; crowding; decentralized congestion control; multi-agent simulation; ant algorithms; swarm intelligence

1. Introduction

A basic principle in ant trail formation is the indirect interaction and communication of ants through the environment by deposited chemical pheromones (*stigmergy*) [1, 6]. It enables ant colonies to make adaptive choices based solely on local information. Together with travel time experiences (e.g. in *Linepithema humile* [2]) or effects of U-turns (e.g. in *Lasius niger* [3]) it leads individual ants to the use of the shortest among several paths to transport food from distant places into their nest.

The stigmergy concept in general allows for a complex collective cooperative behavior of relatively simple agents [7] and has inspired a large number of new algorithms and applications especially in the field of combinatorial optimization problems [8]. This includes shortest path problems [9,11], routing in communication

networks [10] or material flow scheduling on factory floors [12–14].

However, efficient transportation on a system-wide level is not only concerned with shortest paths, but also with traffic assignment and coping with bottleneck situations. An insufficient regulation of traffic flow will usually lead to a congestion of the apparently preferred path. This is one of the most challenging problems in road traffic and routing of data on the internet. Generally speaking, the efficient distribution of limited resources by decentralized, individual decisions is still an open problem in many networked systems.

As will be shown in the following, the investigation of ant behavior can give some hints how to deal with such problems. Therefore, Secs. 2 and 3 will focus on ant traffic and its load-dependent organization, including some analytical investigations. Section 4 will present some new numerical results for a microsimulation of ants based on the social force concept. Finally, Sec. 5 summarizes our results and discusses possible areas of application.

2. Transport and Traffic in Ant Societies

Before we study the optimization of ant traffic under crowded conditions, we shall briefly review some basic properties of the traffic organization in ants.

Mass recruitment in ants has a vital reason: The flow of ants should guarantee an efficient return of food to the nest. This will favor flow maximizing trail systems and traffic organization. However, another objective is the minimization or limitation of round trip times τ (i.e. the time a single ant needs to reach a food source from the nest and to return). Any delay incurred by an ant during its round-trip slows down the amplification mechanism underlying the recruitment process. In situations of competition for food sources even a small decrease in round-trip duration could improve the fitness of a colony against a competitor by rapidly building up a worker force at the food source. A shorter round trip time for the same flow of food return therefore increases the performance and flexibility and, hence, the fitness of an ant colony.

Thus, we expect that ant behavior supports an optimal traffic organization in terms of the flow Φ and round-trip time τ . It is no surprise that these performance criteria also apply to routing algorithms in information networks and that they are meaningful for the transport of goods as well.

In some regards, ant traffic is comparable to vehicular traffic or pedestrian behavior [18]. Laden, nestbound ants experience a slower speed than unladen, outbound ants. Therefore, an ant trail may be congested with large variations in individual speed and agility. However, ants cooperatively driven by shared goals of the colony are different from the rather selfish behavior and distinct individual interests in human transport systems.

In contrast to vehicular traffic, which is organized in unidirectional lanes of traffic, a complete directional separation for outbound and inbound ants is uncommon due to the chemical (chemotaxis-based) trail attraction. Whereas in some ant

species like the wood ant *Formica rufa* [19], some termites [20], army ants [21] or African driver ants [22] forms of unidirectional flow organization can be observed, in most species a separation of opposite flows does not occur. In case of directional traffic, the separation follows certain principles, where either returning or outbound ants use the middle of a trail and the other fraction moves at the outer regions of a trail. In black garden ants (*Lasius niger*) [3] or leaf cutting ants [23] exploiting renewable or permanent resources, no separation is observed at all. This may be explained by the large variations in individual speeds [24].

Presumably, the absence of directional separation leads to collisions between ants moving in opposite directions [23]. But in contrast to larger organisms, ants can stop very abruptly and recover their initial velocity quickly after a frontal collision with another ant.

Theoretically, ant traffic flows can be described in the framework of self-driven many-particle systems [18]. For particle flows in general and for ants on a trail in particular, we have the formula

$$\Phi = w\rho V, \quad (1)$$

where Φ is the flow, ρ the density of ants and V their average speed. w denotes the width of the trail. However, the relation between V and ρ , known as fundamental diagram, is specific to microscopic properties of the driven particles and cannot be determined easily from scratch. Whereas for vehicular traffic and pedestrians a rich body of theoretical work and empirical measurements exists, not much knowledge has been gathered on fundamental diagrams of foraging ants [25–27]. For *Atta cephalotes* (a leaf cutting ant) Burd *et al.* found the empirical speed-density relation [28]:

$$\Phi = w\rho V_m [1 - (w\rho/k_m)^n], \quad (2)$$

where Φ is the flow, V_m represents the maximum average velocity reached under free flow conditions, and k_m , n are empirical parameters estimated in experiments. For *Atta* ants the values of $V_m = 4.04 \text{ cm s}^{-1}$, $k_m = 0.59 \text{ cm}^{-2}$ and $n \approx 0.64$ were confirmed [23,28]. This gives an estimate for the velocity corresponding to the maximum flow of $V = 0.39V_m$.

3. Load-Dependent Traffic Organization in Ants

The basic challenge for any self-organizing traffic optimization is the ability to adapt the routes of partial flows to the overall flow situation in the network. If one link is congested, a certain portion of the overall flow should use an alternative path to reach its destination. The degree of utilization of alternative routes that can be called optimal depends crucially on the capacity and weight of the available alternative paths and the overall flow that has to be served.

To study the organization of ant traffic in a setup with alternative paths and restricted capacity, the binary bridge experiment by Deneubourg *et al.* [29], in which

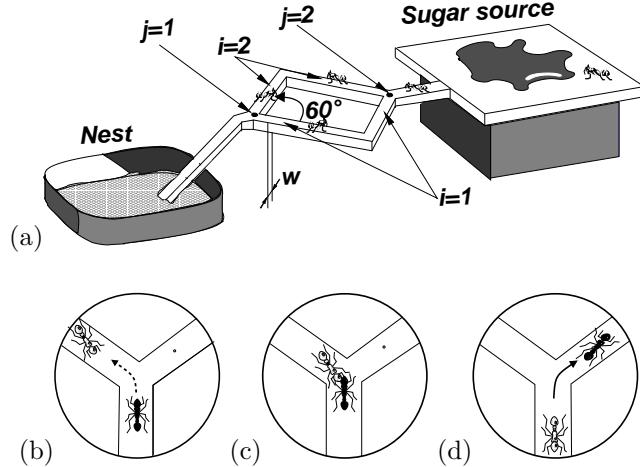


Fig. 1. (a) Experimental setup of a binary bridge experiment. The setup mimics many natural situations, where the physical constraints in the environment restrict the geometry of trails. Moreover, it resembles the situation in technical and economic transport problems, where the geometry of networks is usually predefined by the existing infrastructure. To explore different capacities of the branches $i = 1, 2$, the width w of the branches is varied. (b-d) Typical encounter at one of the choice points $j = 1, 2$. Two ants, coming from opposite directions collide on a branch near the entry point (b). An ant is pushed backwards (c) and redirected towards the other branch (d). (After [4].)

a food source is separated from a nest by a bridge with two alternative, but equally long branches (Fig.1), can be generalized: According to Eq. (1), the capacity of a branch for ant traffic is determined by its width w . Therefore, in a recent experiment with the black garden ant (*Lasius niger*), the widths of both branches has been varied from 10mm to 1.5mm to induce increasing levels of crowding [4].

It was observed that the recruitment dynamics was not significantly influenced by the branch width w , while the ants flow organization changed significantly. For wide branches, most ants used only one of the branches. This spontaneous symmetry breaking can be explained by small initial fluctuations in the pheromone level. During the recruitment process, these initial disturbances are amplified, as a greater number of ants use the trail with the initially or randomly higher pheromone concentration and reinforce it. Surprisingly, the resulting asymmetric trail usage was not conserved for widths $w \leq 6\text{mm}$. Instead, symmetric traffic using both branches occurred.

This symmetry restoring transition can be explained based on head-on encounters of ants traveling in opposite directions [4]. If the width of the bridges is reduced, the branches become crowded and the increased number of encounters of ants reduces their average speed. Therefore, the restriction to one branch occurs only upto a certain critical level of crowding. Above it, head-on encounters at the choice points of a narrow bridge (see Fig.1b-d) trigger the emergence of a symmetric usage of both

branches: When ants entering one of the branches collide with ants from the opposite direction, they are pushed backwards and, with a probability γ of about 60%, they use the other branch after the pushing event at the choice point. While these pushing events were practically not observed on bridges of 10mm width, they occurred frequently on narrow bridges [4]. Loosely speaking, a high congestion of ants on one narrow branch leads to a re-routing of ants through alternative branches.

In the following, we will shortly present a mathematical analysis of this result complementary to what has been published in Ref. [4]. For this, let $\phi_j = \phi_1$ denote the outbound flow of ants from the nest to the food source and $\phi_j = \phi_2$ the nest-bound flow. Moreover, let F_{ij} denote the probability to choose branch $i \in \{1, 2\}$ at choice point $j \in \{1, 2\}$ (see Fig. 1a). For *Lasius niger*, F_{ij} is given by [3]

$$F_{ij} = \frac{(k + C_{ij})^2}{(k + C_{1j})^2 + (k + C_{2j})^2} = 1 - F_{i'j}, \quad (3)$$

where $F_{i'j}$ describes the probability to choose the alternative branch $i' = 3 - i$ at point j . The concentration-dependent choice begins to be effective if the pheromone concentration C_{ij} on branch i behind the choice point j exceeds a certain threshold k . If every ant leaves a fixed amount q of pheromone on its trail and the pheromone decays with a constant rate ν , we can determine the local time-dependent pheromone concentration via the equation

$$\frac{dC_{ij}}{dt} = q[\Phi_{ij}(t) + \Phi_{ij'}(t - T)] - \nu C_{ij}(t). \quad (4)$$

Herein, T is the average time for an ant to get from one choice point to the other and Φ_{ij} the overall flow of ants in direction j (behind choice point j) using branch i . The formula

$$\Phi_{ij}(t) = \phi_j(t)F_{ij}(t)[1 - \gamma a\Phi_{ij'}(t - T)/w] + \phi_j(t)F_{i'j}(t)\gamma a\Phi_{i'j'}(t - T)/w \quad (5)$$

takes into account the probability F_{ij} with which branch i is chosen, but it also considers the effect of pushing events at choice point j to the other branch. $a\Phi_{ij'}(t - T)/w$ is the proportion of ants decelerated on a branch of width w , when the counterflow of ants at choice point $j' = 3 - j$ and time t is $\Phi_{ij'}(t)$. The parameter a is proportional to the interaction time period and the effective cross section (lateral width) of ants. γ denotes the probability of being pushed to the other branch in cases of encounters, i.e. the re-routing probability. One interesting question to be addressed later is, whether the empirically found value $\gamma = 0.57$ is optimal due to evolutionary pressure and whether the optimal value depends on the respective scenario.

Due to the normalization $F_{ij}(t) + F_{i'j}(t) = 1$, we have the relation $\Phi_{ij}(t) + \Phi_{i'j}(t) = \phi_j(t)$, i.e. $\phi_j(t)$ is the overall flow behind choice point j , as demanded. Furthermore, in the stationary case we find $dC_{ij}/dt = 0$, $F_{ij}(t - T) = F_{ij}(t) = F_{ij}$, $\Phi_{ij}(t - T) = \Phi_{ij}(t) = \Phi_{ij}$, and $\phi_j(t - T) = \phi_j(t) = \phi_{j'}(t) = \phi$ (as the nestbound flow and the outbound flow should be equal). This implies

$$C_{ij} = \frac{q}{\nu}(\Phi_{ij} + \Phi_{ij'}) = C_{ij'} = \dots = \frac{q\phi}{\nu}[\gamma a(\Phi_{i'j} + \Phi_{i'j'})/w + 2F_{ij}(1 - \gamma a\phi/w)] \quad (6)$$

and

$$F_{ij} = F_{ij'} = 1 - F_{i'j} = 1 - F_{i'j'}, \quad (7)$$

because of $C_{ij} = C_{ij'}$ for all i [cf. Eq. (3)]. Additionally, with the definitions

$$\frac{C_{ij} + C_{i'j}}{2} = \frac{C_{ij'} + C_{i'j'}}{2} = C \quad \text{and} \quad \frac{C_{ij} - C_{i'j}}{2} = \frac{C_{ij'} - C_{i'j'}}{2} = D \quad (8)$$

one can show

$$C_{ij} = C_{ij'} = C + D, \quad C_{i'j} = C_{i'j'} = C - D \quad (9)$$

[see Eq. (6)] and

$$\Phi_{ij} + \Phi_{ij'} = \frac{\nu}{q}(C + D), \quad \Phi_{i'j} + \Phi_{i'j'} = \frac{\nu}{q}(C - D). \quad (10)$$

A detailed calculation gives

$$C = \frac{C_{ij} + C_{i'j}}{2} = \frac{q}{2\nu}(\Phi_{ij} + \Phi_{ij'} + \Phi_{i'j} + \Phi_{i'j'}) = \frac{q\phi}{\nu} \quad (11)$$

and, using relation (6),

$$\begin{aligned} D &= \frac{C_{ij} - C_{i'j}}{2} \\ &= \frac{q\phi}{2\nu}[\gamma a(\Phi_{i'j} + \Phi_{i'j'} - \Phi_{ij} - \Phi_{ij'})/w + 2F_{ij}(1 - \gamma a\phi/w) - 2F_{i'j}(1 - \gamma a\phi/w)] \\ &= -\gamma a\phi D/w + \frac{q\phi}{\nu}(1 - \gamma a\phi/w) \frac{(k + C_{ij})^2 - (k + C_{i'j})^2}{(k + C_{ij})^2 + (k + C_{i'j})^2} \\ &= C(1 - \gamma a\phi/w) \frac{(2k + C_{ij} + C_{i'j})(C_{ij} - C_{i'j})}{(k + C + D)^2 + (k + C - D)^2} - \gamma a\phi D/w \\ &= \frac{2C(1 - \gamma a\phi/w)(k + C)D}{(k + C)^2 + D^2} - \gamma a\phi D/w. \end{aligned} \quad (12)$$

This allows one to calculate all values of C_{ij} , Φ_{ij} , and F_{ij} with the above formulas. We find

$$C_{ij} = C_{ij'} = \frac{q\phi}{\nu} + D \quad \text{and} \quad C_{i'j} = C_{i'j'} = \frac{q\phi}{\nu} - D, \quad (13)$$

with

$$D = 0 \quad (14)$$

or

$$\begin{aligned} D^2 &= \frac{2C(1 - \gamma a\phi/w)(k + C)}{1 + \gamma a\phi/w} - (k + C)^2 \\ &= C^2 - \frac{k^2 + \gamma a\phi k^2/w + 4\gamma a\phi k C/w + 4\gamma a\phi C^2/w}{1 + \gamma a\phi/w} \\ &= \frac{q^2\phi^2}{\nu^2} - \frac{k^2 + \gamma a\phi(k + 2q\phi/\nu)^2/w}{1 + \gamma a\phi/w}. \end{aligned} \quad (15)$$

As long as $D^2 > 0$, the stable stationary solution is characterized by an asymmetric usage of the alternative branches with $C_{ij} = \frac{q\phi}{\nu} \pm \sqrt{D^2}$ and $C_{i'j} = \frac{q\phi}{\nu} \mp \sqrt{D^2}$. If $\gamma = 0$, $D^2 > 0$ is fulfilled for $q\phi/\nu > k$. For $q\phi/\nu < k$, the traffic flow is too low to generate a certain preference, as the pheromone concentration is not high enough to be noticed. Then, the analytical solution collapses to $D = 0$, resulting in a symmetric usage of both branches, as ants choose between both branches at random.

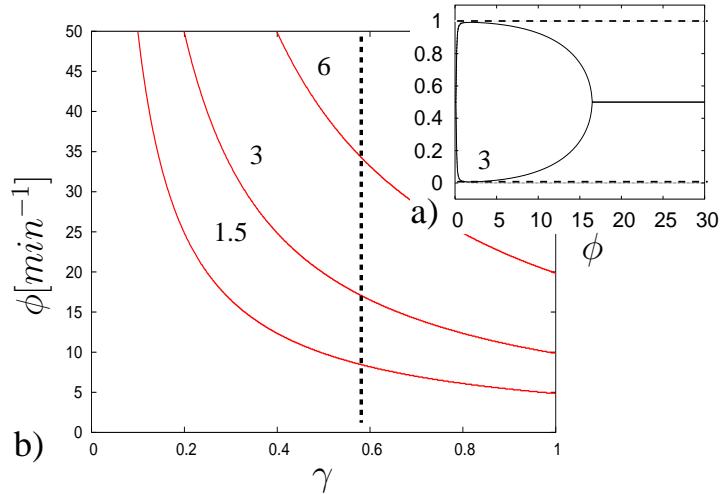


Fig. 2. Analytical results according to Eq. (13) for the parameter values $q = 1$, $k = 6$, and $\nu = 1/40\text{min}^{-1}$. (a) Fraction of the overall flow on each branch in the stationary state, when both branches have a width of 3mm and the value of the re-routing probability γ is 0.57 as observed for *Lasius niger*. The dashed curve shows the solution in the absence of pushing ($\gamma = 0$), where the model predicts asymmetrical traffic for *any* flow above a very low value of the overall flow ϕ . (After [4].) (b) Threshold of the symmetry-restoring transition as a function of the overall flow ϕ of ants and the re-routing probability γ for branch widths of $w = 1.5\text{mm}$, 3mm, and 6mm. The value $\gamma = 0.57$ found for *Lasius niger* is indicated by the dashed line.

While the asymmetric solution is maintained also for high flows, if no pushing is considered (Fig.2a), the situation changes considerably, when a finite re-routing probability $\gamma > 0$ is taken into account: Above a certain critical threshold $\phi_S(w, \gamma)$ of the flow, where $D^2(\phi_S)$ becomes zero again, a symmetric stationary assignment of traffic to both branches is observed, which is caused by pushing events. The critical flow ϕ_S , at which this symmetry-restoring transition takes place, is a function of the width w of the branches and the re-routing probability γ (Fig.2b).

4. Multi-Agent Simulation of Ant Traffic

As the pushing mechanism is based on random encounters in the area of the decision points only (the nodes of the trail network), it is not clear at all that these would be representative for the congestion level on the subsequent part of the respective branch, i.e. the whole link. Therefore, it is hard to imagine that a mechanism as simple as pushing, based on local interactions, would lead to an optimal traffic organization. Nevertheless, it could be shown that the value $\gamma = 0.57$ of the re-routing probability caused the usage of an additional branch shortly before the capacity of a single branch was reached [4]. Was this just accidental, or is the value of γ carefully chosen, e.g. by evolutionary optimization?

In order to find out whether optimality would also be found for other, more complicated scenarios, we have carried out computer simulations (see Fig. 3). Ants were simulated assuming a distribution of velocities and times at which a network with alternative paths (branches) was entered. Their motion was specified by a social force model [35, 36], according to which ants had desired directions (given by the endpoints of network links) and repulsive interactions. The encounters of individual ants were determined, and the probability of re-routing due to a collision near the choice points was given by the parameter γ . Alternatively, with a probability of $1 - \gamma^{dt}$, where dt denotes the time discretization of the numerical integration method, we allowed ants to climb over each other.

Furthermore, in our model we assumed that, once an ant had decided to walk in one direction on a specific link (branch), it would continue in this direction up to the endpoint of this link. That is, we neglected U-turns due to interactions. When an ant reached a node (choice point), the next destination was chosen among all connected links that did not turn more than 90 degrees compared to the ant's previous direction. Stronger turns (apart from U-turns) were only considered, if there were no other links available. The choice probabilities were assumed to be proportional to the squares of the pheromone concentrations on the respective links. These were increased by 1 each time an ant passed. However, before, the pheromone concentrations were multiplied by a certain decay factor in each time step dt .

Microsimulations based on this simple multi-agent model (and similar ones) can reproduce the analytical results in Sec. 3 quite well (see, e.g., Ref. [4]). Moreover, they allow one to study scenarios in which

- the branches have different width and/or lengths,
- there are more than two alternative branches, or
- parameters such as the re-routing probability are chosen different from the empirical value.

This facilitates to study the system performance with and without the pushing mechanism and to relate it to the empirically observed value of γ .

It has been found that the collision-based adaptive routing mechanism can be called optimal. It seems not only to optimize the flow, but also the round-trip time

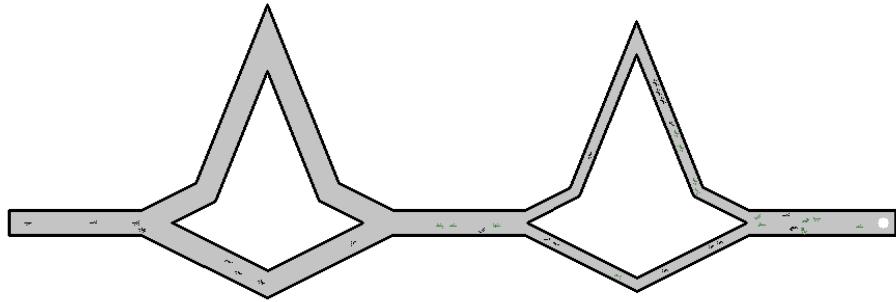


Fig. 3. Simulation setup for a binary bridge scenario. To explore the effect of different capacities of the two alternative branches $i = 1, 2$, the width w_i of the branches is varied. A video film of this scenario clearly shows that the ants mainly use the shorter of two wide branches, while they use both narrow branches (see <http://www.trafficforum.org/ants>).

by limiting the density-related speed reduction. Figure 4 shows the averaged round trip time τ of ants vs. the reached overall flow ϕ as characteristic performance measures for the quality of a routing algorithm. The binary bridge is assumed to be symmetrical, and the parameters applied in the simulation correspond to the findings for *Lasius niger*. The region where the capacity of one branch is reached, is characterized by a sharp increase in the round trip time. Our simulations indicate that, after the symmetry-restoring transition to the usage of both branches, which occurs for heavy traffic with $\gamma > 0$, the available capacity for returning food is doubled compared to a hypothetical scenario without direct interactions and without pushing ($\gamma = 0$).

Our simulations also substantiate positive effects of pushing ($\gamma > 0$) in terms of reduced round trip times. For the same round-trip time, the interaction-based sensitivity to the level of crowding allows a much higher overall flow. In other words, the observed traffic organization guarantees shorter round trip times for the same flow of food return compared to hypothetical scenarios without pushing.

To guarantee the optimality of traffic assignment for a wide range of flows, the value of γ is crucial. It governs the transition point and balances the influence of the pheromone-driven behavior and the congestion-related re-organization of flows. For low traffic volumes, the beneficial properties of pheromone-guided trail-following behavior should dominate. Especially for paths of different path lengths, the majority of ants should be engaged in the shortest path.

We expect that there is an optimal value of γ between 0 and 1: On the one hand, a high value of γ would result in re-routings to longer paths, which is not necessary if the capacity of the shortest path is not exhausted. However, in very crowded situations, ants should use additional paths, even if they are longer. The transition

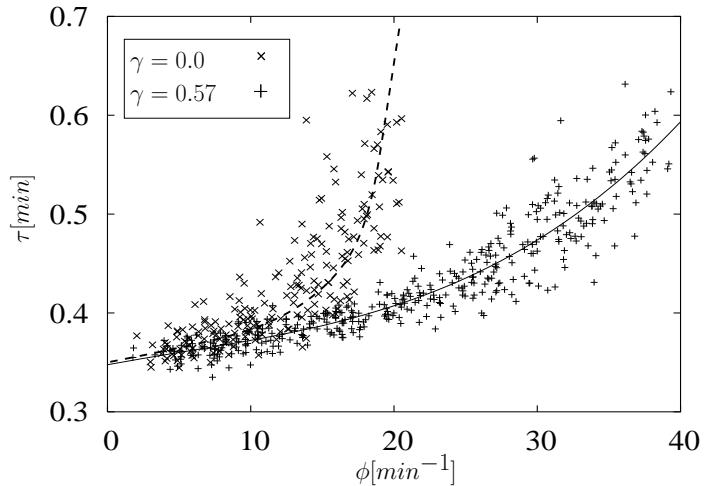


Fig. 4. Results of Monte Carlo simulations depicting the round trip time vs. the overall flow for the binary bridge experiment, taking into account pushing (+) or not (x). The curves are fitted as guide for the eyes, while the dashed curve represents the case $\gamma = 0$, the solid line corresponds to simulations considering pushing.

A symmetric binary bridge with $w = 3\text{mm}$ of both branches was simulated for different flows ϕ . For each data point (representing the result of one Monte Carlo simulation), the values of the round trip time τ were averaged over 4 minutes after the steady state was reached. We used the same parameters as in Fig.2. The empirically observed value of $\gamma = 0.57$ was applied to consider the effect of pushing events. Note that pushing allows to reach higher flow values, as the use of additional branches provides additional capacity.

to this behavior should be driven by the utilization of the available capacity of the preferred branch. On the other hand, if the value of γ is too small, a transition due to pushing never takes place (see Fig. 2b) or only, after a undesirable level of congestion has been produced. From these considerations we conclude that the optimal value of γ would trigger the symmetry-restoring transition just before the capacity of a branch is reached, as has been actually observed [4].

For asymmetric binary bridges with branches of the same width but different length, the interrelation between the optimal traffic distribution and the parameters of the local feedback through physical interactions near the choice points can be discussed also for flows below the capacity of a single branch (Fig.5): Due to the pheromone-guided behavior, the shorter branch is used more frequently, but with increasing γ , a larger and larger portion of flow utilizes the longer branch. This effect is observed even if the critical flow for symmetry restoring is not reached. For geometrically asymmetric binary bridges, the symmetry restoring transition is characterized by a continuous function

$$\Phi_{1j}/\Phi_{2j} \sim l_1/l_2. \quad (16)$$

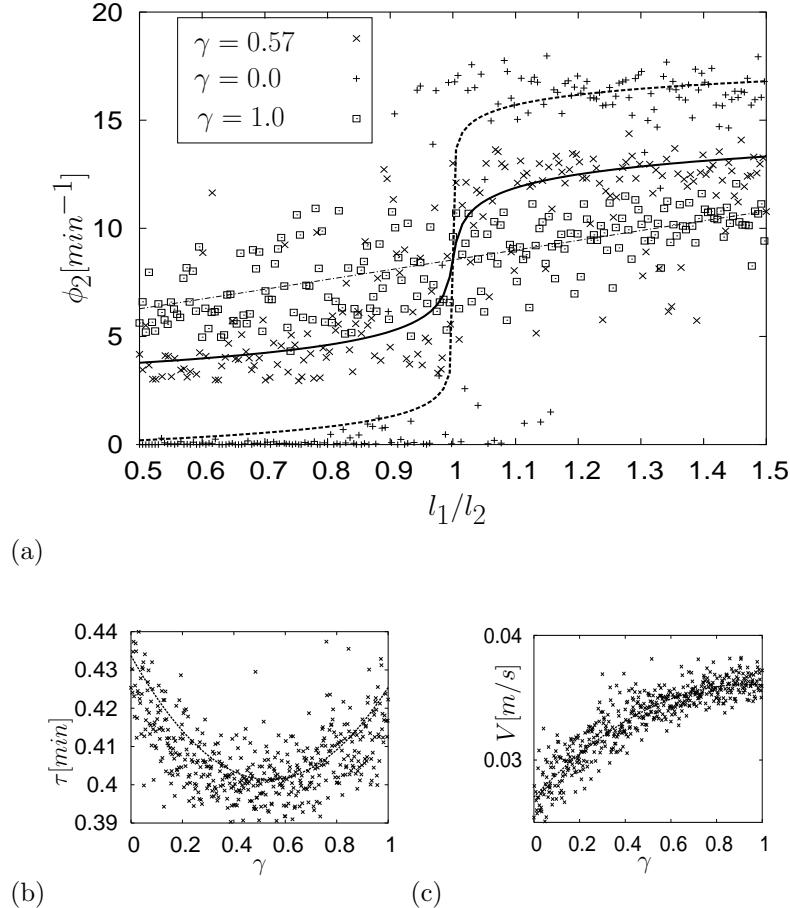


Fig. 5. Results of Monte Carlo simulations for binary bridges with different length of the alternative paths. An overall flow $\phi = 15min^{-1}$ was applied. The length of branch 1 was fixed ($l_1 = 10cm$), while the length l_2 of branch 2 was varied. The simulations were carried out for different values of the re-routing probability γ . Each point represents an average over 10 minutes of simulated ant traffic. (a) Fraction Φ_2 of the overall flow using branch 2 vs. l_1/l_2 for $\gamma = 1$ (every encounter results in a re-routing), $\gamma = 0.57$ (the value observed in *Lasius niger*) and without pushing ($\gamma = 0$). For all values of γ , the inequality $\Phi_2 < \Phi_1$ for $l_2 > l_1$ reflects the pheromone-driven preference for the shortest path. The curves are fitted as guide for the eyes: While the dashed curve corresponds to $\gamma = 0.0$, the solid curve represents $\gamma = 0.57$ and the dashed-dotted curve $\gamma = 1.0$. (b) Averaged round-trip time τ vs. the re-routing probability γ for $l_1 = 10cm$, $l_2 = 20cm$ and $\phi = \Phi_{11} + \Phi_{21} = 15min^{-1}$. In this situation, the experimentally observed value of $\gamma = 0.57$ is the optimal value to obtain a minimal round-trip time. A second order polynomial was fitted to the simulation results as guide for the eyes. (c) Average velocity V vs. γ . The same parameters as in (b) were applied. This plot shows that the re-routing probability γ does not optimize the velocity, which is maximal if nearly equal flows are assigned to both branches.

Thus, the flows beyond the transition point ϕ_S are directly proportional to the lengths of the alternative paths, but below the transition point, a jump in the assignment occurs for the symmetric binary bridge. In Fig. 5a, we have $\phi < \phi_S(\gamma)$ for $\gamma = 0$ and $\gamma = 0.57$, whereas $\phi > \phi_S(\gamma)$ if $\gamma = 1$.

The optimal assignment depends on the relative length of both branches and the fundamental diagram of traffic. In certain situations we have found numerically that a re-routing probability of $\gamma = 0.57$ provides the optimal behavior in terms of flows and round-trip times (Fig. 5b) even for sub-critical flows ($\phi < \phi_S$). However, for other geometries of the path network, we have found that other values of the re-routing probability ($0.5 < \gamma < 0.8$) can improve the traffic assignment.

In the same way as for binary bridges with asymmetric length, traffic organization is also sensitive to capacity. If two branches i with different widths w_i are simulated, the flows in the heavy traffic case (after symmetry restoring) adapt to the capacity of the available branches according to

$$\Phi_{1j}/\Phi_{2j} \sim w_2/w_1. \quad (17)$$

In general, the results obtained in simulations with different capacities are similar to the traffic organization in the case of two branches with equal capacities, but different lengths.

Our microscopic simulations support the conjecture that the discovered collision-based traffic organization principle in ants generates optimized traffic for a wide range of conditions. It optimizes the utilization of available capacities and minimizes round-trip times. Simulation results for a symmetrical bridge with four branches are shown in Fig. 6. It can be seen that the majority of ants uses one of the large branches and, within both branches, the majority of ants uses one of the small branches. Which of the branches is preferred basically depends on random fluctuations or the initial conditions. When the overall ant flow is increased, the ant flows on the branches become more equally distributed. Above a certain overall flow, the usage of the bridge is completely symmetrical, as for a bridge with two branches. On a logarithmic flow scale, it appears that we do not have a *discontinuous* transition from the use of one small branch to the use of two small branches and another transition to the use of three or four branches. However, the sharpness of the transition is certainly a matter of the choice of parameters.

5. Summary and Outlook

In this article, we have discussed a mechanism of traffic organization in ants that, on the one hand, establishes the shortest connection based on pheromone attraction, but on the other hand, guarantees the usage of additional capacities by collision-based re-routing. In the future, such mechanisms are likely to have interesting applications in the tradition of ant algorithms and swarm intelligence: For example, if we have a number of machines with comparable production functions, it will be most profitable to use one machine only when the production load is small, and to add

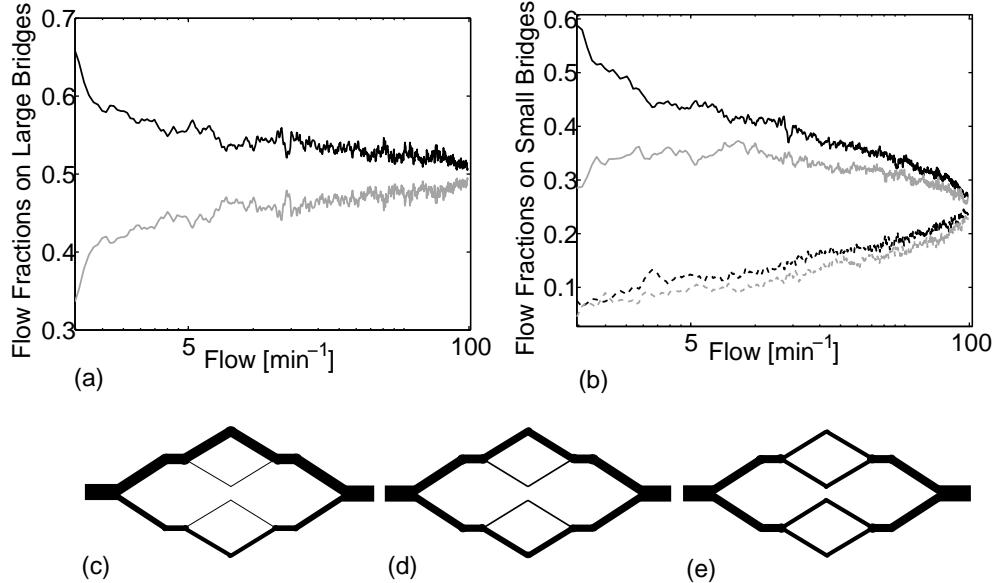


Fig. 6. Simulation results for a completely symmetrical bridge with two large branches that are again subdivided into two small branches each. The overall width of the bridge (i.e. all branches) is the same everywhere. (a) Fraction of ants on the two large branches as a function of the overall ant flow ϕ . (b) Fraction of ants on the four small branches. The lower figures give a schematic representation of the distribution of ant flows over the large and small branches (c) at very low flows, (d) at medium flows and (e) at large flows.

further machines when the load exceeds certain thresholds. Note that the optimal thresholds, just as observed for ants, are usually somewhat below the production capacity of a machine, as it will in many cases not be advisable to go to machine limits for reasons of costs, reliability, maintenance, etc. Machine parameters such as load-dependent cycle times play a role similar to the lengths of alternative paths in ant systems.

Altogether, cost-efficient load-balancing of machines or transportation capacities can be a difficult issue, and interaction-based ant algorithms seem to be a promising approach, particularly as they do not require central control with the potential of information losses and delays or decision conflicts. Instead, they are based on simple local decision rules.

Applications to the routing of traffic are also an interesting subject. Clearly, congestion can only be avoided as long as there are enough alternative paths over which traffic can be distributed. If this capacity is not available, the transferability of our ant model is limited. However, it appears that congestion in data and vehicle traffic could be reduced by better routing algorithms. The interaction-based approach by ants would be promising here as well, although generalizations will be needed due to multi-origin-multi-destination flows that are far from being stationary. One

interesting research direction would be the investigation of decision functions different from Eq. (3). Another focus could be the extension of the interaction concept, replacing collisions by passing messages between oppositely moving flow directions. In this way, information about the traffic volume on different (directed) links could be passed by local interactions similarly to ad-hoc networks based on transversal hopping, which have been suggested for inter-vehicle communication [37]. Thereby, it should be possible to establish a decentralized algorithm for system-wide travel time or congestion information (cf. Ref. [38]), which could be the basis for optimal (re-)routing decisions.

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